

THE KING'S SCHOOL, CANTERBURY



SCHOLARSHIP ENTRANCE EXAMINATION

February 2011

MATHEMATICS 1

Time: 45 minutes (plus reading time)

Use the reading time wisely; gain an overview of the paper and start to think of how you will answer the questions.

Do as many questions as you can (clearly numbered) on the lined paper provided. Clearly name each sheet used. You are encouraged to attempt these questions in order.

The questions are not of equal length or mark allocation. Make sure you avoid spending too much time on any one question; don't get bogged down! Move on quickly if you get stuck. The paper is quite long; you are not necessarily expected to finish everything.

Some of the later questions are more difficult, but not necessarily longer. Some questions are designed to test your ability to work with unfamiliar ideas, or familiar ones with a twist. Don't give up!

*You are expected to use a calculator where appropriate, but also you must show **full and clear working**, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers. In fact, merely writing down an answer might score few marks.*

Complete questions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.

This paper has nine questions.

1 Starbucks coffee houses have introduced a new largest size of coffee, the Trenta. This is a 55% increase on the previous largest coffee size, the Venti, which measures 591 ml.

How big is a Trenta (give your answer to the nearest millilitre)?

2 The oil company Shell made a profit of \$5.7 billion in the last three months of 2010.

A recent headline in the Guardian newspaper claimed that this amounted to £1.6 million per hour in profits during those three months.

Show, with careful working, that this claim is justified. [Use an exchange rate of \$1.65: £1.00, and note that a billion is 1 000 000 000.]

3 (a) Simplify as far as you can:

$$\frac{78p^3q^7}{12q^2p^8}$$

(b) Factorise

$$72x^3 + 18x$$

4 Last month Cadbury's Dairy Milk bar was reduced from 140g to 120g but the price stayed at 99p.

(a) What percentage reduction in weight is this?

(b) What is the percentage effective price increase (since we are getting less chocolate)? [Hint: think in terms of cost per gramme]

5 Solve the following equations:

(a) $7(2x + 3) = 84$

(b) $7 - \frac{x}{8} = 11$

(c) $2(3 + x) - (4 - x) = 29$

6 In this question the **reciprocal** of n is

$$\frac{1}{n}$$

A number is called a **Giuga Number** if, when you find the sum of the reciprocals of the prime factors of the number, and then subtract the reciprocal of the original number, you get an answer of 1.

Here are a couple of examples.

30 is a Giuga Number, since

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{5} - \frac{1}{30} = 1$$

Also, 858 is a Giuga Number, since

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{11} + \frac{1}{13} - \frac{1}{858} = 1$$

Show (with enough working) that 1722 is also a Giuga Number.

7 (a) Solve the following simultaneous equations in x and y .

$$2x - 5y = 13$$

$$x + 2y = 11$$

(b) Using your answers to (a) **write down** the solutions (for x and y) to the following pair of simultaneous equations.

$$2x^2 - 5y^2 = 13$$

$$x^2 + 2y^2 = 11$$

8 In the UK VAT (Value Added Tax) is now set at 20% (so, when you buy something in a shop, the price you pay has had the 20% VAT added already).

(a) Explain carefully what is wrong with the advert below (seen last month on the Tesco supermarket website).



Suppose I buy a bicycle in this sale for £180.

(b) What was the original selling price before the 20% reduction?

(c) What should the sale price have really been if Tesco had **correctly** removed the VAT from this original selling price?

(d) Compare your answer to (c) with the £180 you paid.

9 in 2009 Professor Simon Blackburn worked out a formula for the perfect parallel parking manoeuvre for a car parking between two other parked cars on a straight road.

Suppose we call X the amount by which my parking space must be longer than my car [In this question all lengths are in metres].

Then

$$X = \sqrt{(r^2 - l^2) + (l + k)^2 - \left(\sqrt{r^2 - l^2} - w\right)^2} - l - k$$

<i>What do the letters mean?</i>	<i>In this question</i>
r is the radius of my car's kerb-to-kerb turning circle	$r = 5.55$
l is my car's wheel-base (the distance between the centres of the front wheel and the corresponding back wheel)	$l = 2.6$
k is the distance from the centre of the front wheel to the front of the car	$k = 1.4$
w is the width of one of the parked cars: the one near the front of my car once I've parked.	$w = 1.7$

Using your calculator, use the formula given above to work out carefully what the value of X must be, ie the extra length of the parking space needed. Write down all the numbers on your calculator display when you have finished.

[Note: you do not need to understand where the formula comes from – just be able to substitute the values and work out the answer.]

END OF PAPER